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# On a Throw-Testing Machine for Reversals of Mean Stress

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# IV. *On a Throw-Testing Machine for Reversals of Mean Stress.*

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## PREFACE.

ALTHOUGH this research is a joint undertaking, I wish to point out that except for the idea and general design of the apparatus all the work in carrying out the design has been done by Mr. SMITH, who has made all the calculations and superintended the execution of all the work which he has not executed himself. This undertaking occupied some two years, being not only novel but also approaching fundamental limits which, if passed, would have wrecked the undertaking. He has also made all the tests. Thus whatever success we have had is entirely owing to his knowledge, skill, and perseverance. In saying this, I do not wish to imply that I have not taken great interest in the work, for, on the contrary, I have watched it with interest and admiration, particularly the acumen he has shown in arranging his tests and interpreting the results, by which he has obtained evidence of two general laws which had not hitherto been suspected, one being that under a given range of stress the number of reversals before rupture diminishes as the frequency increases, and the second that the hard steels will not sustain more reversals with the same range of stress than the mild steels when the frequency of the reversals is great.

OWENS COLLEGE,
*February* 19*th*, 1902.

OSBORNE REYNOLDS.

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### *Historical Summary.*

In 1860, Sir W. Fairbairn, using a riveted girder, carried out a series of experiments, which seem to be the first recorded experiments on Repeated Stress.

From 1860 to 1870, Wöhler carried out his laborious and valuable researches on the Fatigue of Wrought Iron and Steel. From his results published in the 'Zeitschrift für Bauwesen,' Berlin, the following important points may be deduced:—

(1) That these materials (wrought iron and steel) will rupture with stresses much *below* the statical breaking stress, if such stress be repeated a sufficient number of times.

(2) That within certain limits, the *range* of stress, and not the maximum stress, determines the number of reversals necessary for rupture.

(3) That as the range of stress is diminished, the number of repetitions for rupture increases.

(4) That there is a *limiting range* of stress for which the number of repetitions of stress for rupture becomes infinite.

(5) That this limiting range of stress diminishes as the maximum stress increases.

Wöhler conducted his experiments on bars of wrought iron and steel, subjecting them to torsional stress, bending stress, equal and opposite bending stresses, and direct tension, with repetitions ranging from 60 to 80 per minute.

In 1874, Spangenberg repeated Wöhler's experiments, using Wöhler's machines, and obtained similar results, also published in the 'Zeitschrift für Bauwesen.' In 1874 also, Gerber, in the 'Zeitschrift für Baukunde,' München, suggested the following formula, as representing the results of Wöhler's experiments:

$$f\,(\text{max}) = \tfrac{1}{2}\Delta + \sqrt{(f^2 - n\,\Delta f)},$$

where $f\,(\text{max})$ = the maximum stress,
$f\,(\text{min})$ = the minimum stress,
$f$ = the statical breaking stress,
$\Delta$ = the range of stress $= f\,(\text{max}) \pm f\,(\text{min})$,
and $n$ = a constant.

Accounts of other experiments and theories bearing on this subject, are given by the following:—

Launhardt (Zeitschrift des Architecten und Ingenieur-Vereins, Hanover, 1873).
Lippold (Organ für die Fortschritte des Eisenbahnwesens, Wiesbaden, 1879).
Professor Mohr (Der Civil-Ingenieur, Leipzig, 1881.)

Sir B. BAKER in 1886 gave the results of a series of experiments on iron and steel (*Am. Soc. Mechanical Engineers*), which were obtained with machines similar to WÖHLER'S, the bars being rotated in one set of tests and subjected to bending in the other sets, the repetitions taking place 50 to 60 times per minute.

About the same time BAUSCHINGER* published his important memoir on the "Variation of the Elastic Limits," in which it is shown that when the elastic limit in tension is raised, the elastic limit in compression is lowered, and that by subjecting a material to a few alternations of equal stresses, the elastic limits tend towards fixed positions, in which positions he called them the *natural* elastic limits. BAUSCHINGER then proceeded to explain the results obtained by subjecting material to repeated stresses, by showing that the limiting range of stress coincided with the difference of the two elastic limits.

## *Objects of the Research.*

The present research, which was carried out in the Whitworth Engineering Laboratory of the Owens College, was undertaken at the suggestion of Professor OSBORNE REYNOLDS, who proposed an investigation of "repeated stress" on the following lines:—(1) The stress should be direct tension and compression; and (2) of approximately equal amounts, such tension and compression being obtained by means of the inertia force of an oscillatory weight; (3) the rapidity of repetitions should be much higher than in the experiments of WÖHLER, SPANGENBERG, BAUSCHINGER, and BAKER, in fact, ranging as high as 2000 reversals per minute. The importance of these points will be seen from the following considerations:—

(1) By far the greater number of experiments on "repeated stress" have been carried out on bars subjected to bending, the ordinary formula for stress in a bent bar being used to calculate the stress at breaking, that is, in such experiments it has been assumed that the distribution of stress at the breaking-down point is the same as for an elastic bar. Calculations on this assumption are not expected to give the tensile strength of a material for an ordinary cross-breaking experiment. This difficulty is completely overcome, and no such assumptions are necessary, when the stresses are direct as in the present work. The (direct) stress in a bar of metal could easily be obtained by having one extremity rigidly connected to a part of a machine having a known periodic motion, the other extremity being attached to a known weight.

(2) The tensile stress being in all experiments nearly equal to the compressive stress, the elastic limits would, as shown by BAUSCHINGER, soon be changed to their *natural* positions, and the range of stress for unlimited reversals would be this natural elastic range. If then, the limiting range coincides with the natural range it will be constant whatever the rate of reversals. The author considered this point an interesting one, and it will be found that most of the tests recorded in this paper

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* 'Mittheilungen aus dem Mech. Techn. Laboratorium in München,' 1886.

were carried out in such a way as to find the variation of the limiting range of stress as the rapidity of such reversals increased. The apparatus to be described shortly was most convenient for such enquiry, since both the speed and the oscillatory weight could be easily adjusted.

(3) Quite apart from the point mentioned in the last paragraph, the importance of extending the experiments to high speeds—in view of the extensive use in recent times of high speed machinery—is too obvious to need comment.

Short specimens of small diameter (see fig. 4), had to be used throughout, otherwise the apparatus would have been inconveniently heavy, and for this reason any subsequent work on the statical strength and the elastic properties of specimens which had been subjected to repeated stress, could not be done.

## *Method of Applying the Stress.*

A weight is supported vertically by means of the specimen to be tested, and the upper part of the specimen receives a periodic motion in a vertical direction by means of a crank and a connecting rod. The inertia of this weight gives a tension at the bottom end, and a compression at the top end of the stroke, the change from tension to compression being gradual. The specimen and parts are guided by suitable bearings placed in a vertical direction. The motion was made vertical in order to reduce the friction of the bearings to a minimum. The stresses can be changed by varying the diameter of the specimen, the load, and the speed of revolution of the crank. In order to enable one to calculate the stresses in the specimen, the centre of the crank shaft must be at rest, and the crank must move with uniform angular velocity. These conditions are obtained when the crank shaft is driven by a constant turning effort, if the moving parts of the machine are balanced, and if at the same time the total energy of the moving parts is invariable. The apparatus was therefore designed to satisfy these conditions as approximately as possible (see pp. 270 to 272).

## *The Apparatus.*

On examining the drawings (figs. 1 and 2) of the testing machine, which show the working parts, it will be seen to consist of a cast-iron standard having two brass bushed bearings in its upper part. In these bearings a shaft, 4 inches diameter at the front end, and 2 inches diameter at the back end, revolves, driven by a stepped pulley keyed to this shaft at the back part of the machine. The standard is mounted upon a heavy cast-iron bed-plate (weight, 14 cwts.), not shown in the drawings.

The front end of the shaft is cranked, the crank pin being $1\frac{1}{2}$ inch diameter, 2 inches long, and throw $\frac{1}{2}$ inch, and a connecting rod of peculiar form is coupled to the crank pin. One part of this connecting rod gives an oscillatory motion in a vertical direction to the sliding pieces directly below the crank shaft, which pieces

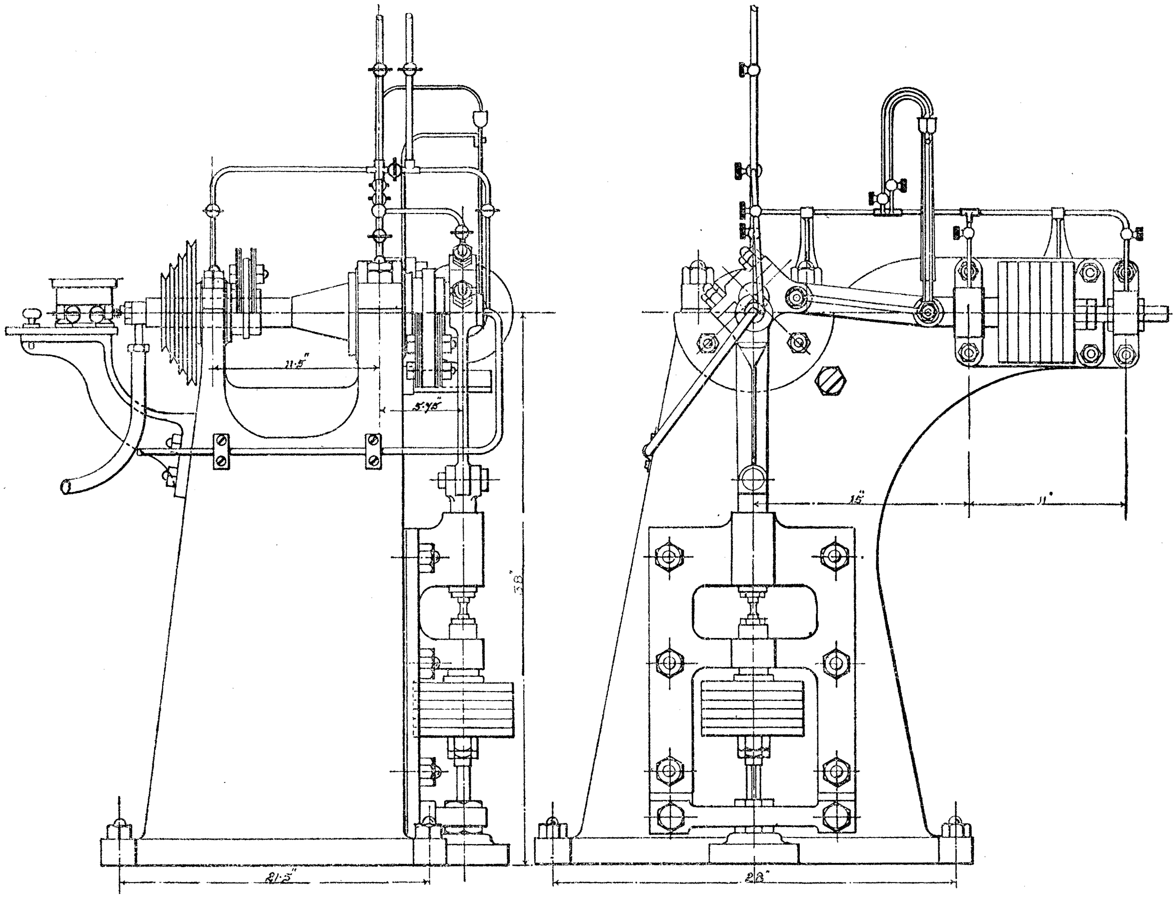

Fig. 1.

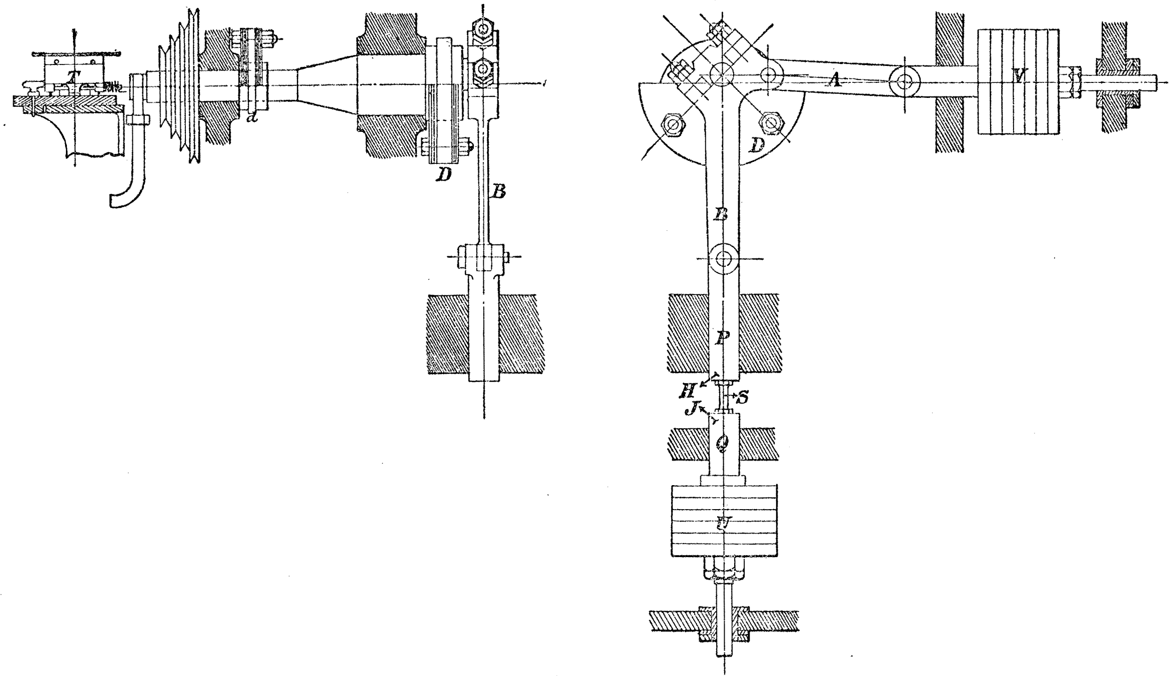

Fig. 2.

include the specimen to be tested. A pin in this connecting rod at the level of the crank shaft gives, by means of a second rod, an oscillatory motion to horizontal sliding pieces, introduced as will be seen later (p. 270) for the purpose of having the energy of the moving parts invariable.

The two parts P and Q of the vertical sliding pieces are connected by means of the specimen S, which is to undergo the test. The chucks H and J for holding the specimen were chased out internally to $\frac{3}{4}$-inch Whitworth thread, and the specimen was locked by means of two lock-nuts, one at each end. The specimen was prevented from rotating by means of a key placed in the lower bearing of the vertical sliding piece, which fitted accurately in a keyway cut in the moving spindle. The lower bearing was bushed to allow of adjustment, and a suitable locking arrangement was provided for it.

All the working parts were well made and exceptionally strong, of mild steel, tensile strength 24 tons per square inch; the pins in the connecting rod were all casehardened and afterwards ground to fit. The greater part of the tool work was done by the author in the College Laboratory.

## *Energy of the Parts.*

The horizontal sliding piece was introduced in order to make the energy of the moving parts constant. Since the vertical connecting rod is 24 times, and the horizontal connecting rod 18 times, the throw of the crank, the motions of both sliding pieces will be very approximately simple harmonic motions, and, as both these pieces receive their motion from the same crank pin, the velocity of one will vary as the *sine*, and the other as the *cosine* of the angular displacement of the crank. The sum of the squares of their velocities will be constant. The kinetic energy of the parts will thus be constant if the total mass moving in the horizontal direction is equal to that moving in the vertical direction.

The masses of the parts were adjusted to satisfy this condition in the following manner:—The connecting rod and the spindles were weighed in the two positions shown in fig. 3.

Firstly, the shorter connecting rod A was supported horizontally and the load on the crank pin weighed; secondly, the longer connecting rod B was supported horizontally and the weight on the crank pin again taken. The masses of these parts were then adjusted until the loads on the crank pin were the same in the two cases.

## *The Balancing of the Machine.*

Having adjusted the masses of the horizontal and vertical sliding pieces, it was now possible to balance these parts by placing a suitable mass diametrically opposite to the crank pin. This balance weight was made in the form of a steel eccentric D (fig. 2), $8\frac{1}{2}$ inches diameter, ·7 inch thick, and throw $1\frac{1}{4}$ inches, and was keyed to the shaft as near as possible to the crank pin.

This arrangement introduces an unbalanced couple in a plane passing through the centre line of the crank shaft and rotating with the shaft, and to balance this couple a smaller eccentric *d* (fig. 2) of diameter $5\frac{1}{2}$ inches, thickness $\frac{1}{2}$ inch, and throw 1 inch was placed near to the far end of the shaft with its centre in the axial plane passing through the crank pin.

So far then—neglecting the obliquity of the connecting rods, which were respectively 24 and 18 times the throw of the crank—the unloaded machine was balanced, and the kinetic energy of the parts was constant.

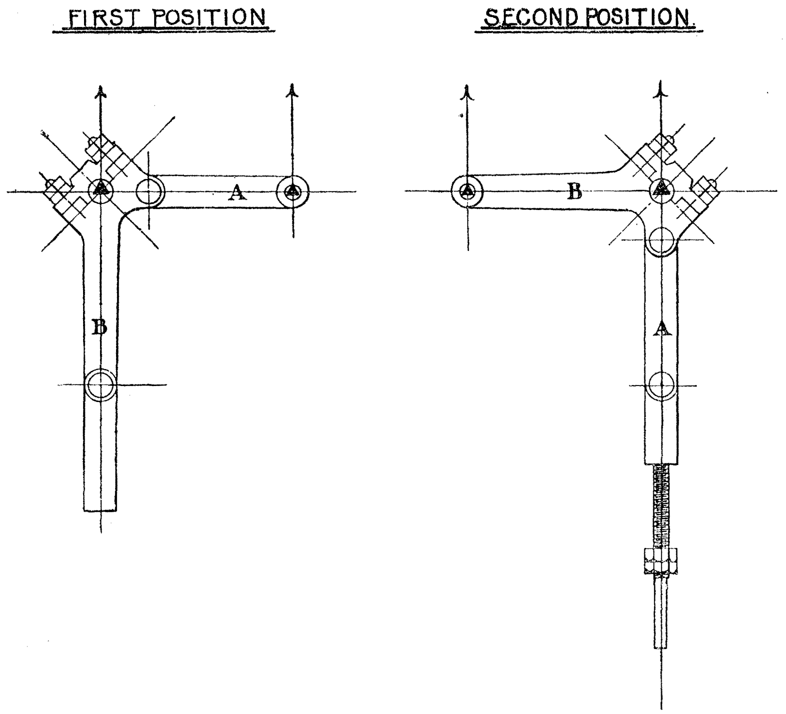

Fig. 3.

In loading the machine the system of weights used was so designed that when one cast-iron weight of $6\frac{1}{4}$ pounds was added to each of the oscillating pieces (U, V, fig. 2), and one semicircular $\frac{1}{16}$-inch steel plate was added to each side of each eccentric balance weight (D and *d*) the machine was still balanced. It was necessary to use the two balance weights to each eccentric (one on each side) in order to keep the plane of the unbalanced force due to each pair in a constant position, that is, in the central plane of the eccentric perpendicular to the shaft. Fig. 2 shows the working parts of the machine fully loaded.

To prevent any vibration from being transmitted to the building when the machine was running unbalanced, and to hold the machine in position, the bed-plate was supported by four spiral springs (made of $\frac{1}{2}$-inch steel of square section), 3 inches diameter, which were fitted into cases specially cast on the under side of the bed-plate for them, their other extremities being sunk into the floor of the laboratory. The machine was mounted on the bed-plate in such a manner as to bring the vertical line of the oscillating piece into the centre line of the bed-plate, and about 3 cwts. of cast-iron was then bolted to the inside of the bed-plate, and its position adjusted so as to bring the surface horizontal.

The machine was driven from the countershaft by means of a $\frac{3}{8}$-inch rope and large stepped grooved pulley, 30 inches diameter, a movable pulley being used to adjust the tension of the driving cord.

At the left side of the bed-plate there was attached a speed indicator designed and constructed by Mr. T. Foster, Mechanical Expert in the Whitworth Engineering Laboratory. The form of the indicator is simple, the speed being indicated by the rise of water in a tall glass tube due to the "centrifugal force" produced by setting it in rotation by means of a spindle driven by the machine. The author is indebted to Mr. Foster, not only for this, but for many valuable suggestions and many excellent pieces of his workmanship during the construction of the apparatus.

A central gunmetal spindle, driven from the crank shaft by a small gut band, has attached to it four radial vanes which revolve in a cylindrical brass box filled with water. A glass tube rising vertically is connected to the lower part of the case, and has a scale attached which is graduated by means of a revolution-counter to measure revolutions per minute. The case containing the water was arranged so that the amount of water used could be accurately regulated. Coloured water was first used, but it was found better to use pure water as the colouring matter was deposited on the tube, and after a few weeks made the taking of readings difficult.

In addition to this, at the back end of the machine a cast-iron bracket is bolted to the standard. This has its upper surface planed, and on it a small table having a revolution-counter (T, fig. 2), attached to it. The table slides on the surface, being guided so as to allow the counter to pass in and out of gear with the end of the crank shaft. A steady pin was used to hold the counter in its different positions.

A lead buffer is used to receive the blow from the vertical oscillatory weight when the specimen breaks. Two cast-iron pieces F and G (fig. 5, p. 279), keep the buffer central and are so arranged that when F is lifted G can be removed. A conical piece of lead is inserted in the centre of the piece F, and is directly under the vertical spindle. The pieces of lead can easily be replaced, and it was found necessary to replace them after every three or four tests.

## *Method of Lubrication.*

A great amount of difficulty was experienced in supplying the oil to the various bearings. A very thick oil was used for the crank pin, and an ordinary machine oil for the other parts. In the final arrangement of the apparatus, the oil was supplied from two large glass vessels supported by brackets fixed to the wall behind the machine. The oil was led by means of two $\frac{3}{8}$-inch brass pipes to the machine, one pipe being arranged to feed into a brass cup at the extremity of the crank pin, the other having a number of branches passing to the various bearings in the upper part of the machine.

The brass cup supplies the oil to the crank pin by centrifugal force, by means of a hole passing along parallel to the centre line of the shaft and then at right angles into the crank pin bearing. The two pins in the horizontal connecting rod were supplied with oil from two vertical pipes having cups to receive the oil at their upper extremities, which pipes are connected together and oscillate about a pin supported by a bracket attached to the frame of the machine. From one pipe the oil is led directly up the centre of the pin connected to the horizontal sliding piece, and from the other the oil is led to the same pin but passes along a pipe attached to the connecting rod to the pin at the other end of the rod.

Sheet iron shields are placed about the revolving parts, and these collect the oil thrown off, carry it down the vertical rod, and allow it to drain into a cup and pipe for carrying the oil so drained to the lower pin of this rod. The bearing of the vertical sliding piece, above the load, receives the oil drained from the upper parts of the machine; the bearing below the load receives the oil from passages cut in the sliding spindle through which the oil passes on its downward course. The crank shaft bearings and the horizontal sliding piece bearings have each a separate oil supply pipe.

A sheet iron trough is inserted between the frame of the machine and the bed-plate to catch the oil. The oil is taken out of the troughs, passed through a filter, and again used. The thick oil was used for nothing but the crank pin for some time, but, owing to the mixing of the oils in the lower trough, the oil all became gradually of a heavy variety, so that in about a few months the same oil was used for all the bearings.

The machine, it was found, worked well after a few months' running, but on changing from slow to high speeds, or *vice versâ*, a little trouble was always experienced owing to the bearings heating. To help to keep the crank shaft bearings cool,—as it was these bearings which heated most easily,—a hole was drilled right through the whole length of this shaft, and a brass junction was specially made for the back end; to this junction an india-rubber pipe conveying a stream of water was connected. The water passed along the hole in the crank shaft to the front of the machine, where, on passing out, it was received by a pipe of larger diameter, through which it was drained away.

### *Determination of Stress.*

If W is the weight below the specimen, R the radius of the crank, L the length of the vertical connecting rod, $\omega$ the angular velocity of the crank pin, and $\Delta$ the area of the specimen, then the compressive stress in the specimen at the upper end of the stroke is equal to

$$\frac{W\omega^2 R}{g\Delta}\left(1 - \frac{R}{L}\right) - \frac{W}{\Delta},$$

and the tensile stress at the lower end of the stroke to

$$\frac{W\omega^2 R}{g\Delta}\left(1 + \frac{R}{L}\right) + \frac{W}{\Delta}.$$

The range of stress is equal to the sum of these, or is equal to

$$2 W \omega^2 R \div g\Delta.$$

The values of W, the weights of the vertical loads used below the specimen, were determined to one hundredth of a pound, and were as follows:—

| | |
|---|---|
| Spindle and lock-nut . . . . . . . . . | 6·15 pounds. |
| ,, lock-nut, and one weight . . . . . | 12·42 ,, |
| ,, ,, ,, two weights . . . . . | 18·69 ,, |
| ,, ,, ,, three ,, . . . . . | 24·96 ,, |
| ,, ,, ,, four ,, . . . . . | 31·23 ,, |
| ,, ,, ,, five ,, . . . . . | 37·50 ,, |
| ,, ,, ,, six ,, . . . . . | 43·77 ,, |

The error in the determination of the stress due to the maximum error in the estimation of these weights would not in any case exceed ·3 per cent.

The throw of the crank was measured to a ten-thousandth of an inch, the value obtained being ·5067 inch. The maximum error in this measurement would not affect the stress by more than ·1 per cent.

The areas of the specimens were determined by finding their diameters by means of an ordinary micrometer gauge which was graduated to ten-thousandths of an inch. Assuming that the greatest error in actual measurement would not be more than three ten-thousandths of an inch, then the error from this cause for a specimen $\frac{1}{4}$ inch diameter would not exceed ·25 per cent.

It is thus seen that the errors incurred in the estimation of W, R and Δ are negligible.

There are three sources of errors in the estimation of the angular velocity. They are—

(1) The variation due to fluctuation in the energy of the parts;

(2) The variation due to fluctuation of the velocity of the engine in a cycle; and

(3) The variation due to the fluctuation of velocity over a long interval arising from the difficulty of regulating the motive power.

(1) To determine the extent of the variation due to the first cause, the curves of displacement for the oscillating parts were carefully drawn to a large scale, and the harmonics of the motion found by the usual graphical method. The harmonics were, however, very small, and as the kinetic energy of the reciprocating parts is only about $\frac{1}{30}$th of the total kinetic energy of the rotating parts, it is evident that the fluctuation of energy of the parts could only introduce infinitesimal fluctuations in angular velocity.

(2) The fluctuations of velocity of the engine in a cycle, loaded as it was by a heavy fly-wheel and rope pulley, and connected with a long line of shafting having a large number of heavy pulleys attached, are also negligibly small. Thus the only real difficulty in eliminating errors in the measurement of $\omega$ was found in overcoming the secular variations of velocity.

(3) In the first series of experiments, the machine was driven by a Crossley's oil engine of three horse-power, but the fluctuations of velocity were not small enough, even when the engine was working with full load, for this mode of driving to be considered satisfactory. Although the author was not content with the results obtained under these conditions, yet, for the sake of comparison, the results of one set of 20 tests are given in this paper.

The machine was finally driven by the low-pressure engine of the triple expansion experimental engines. These engines are described in a paper on 'The Mechanical Equivalent of Heat,' by Professor OSBORNE REYNOLDS and W. H. MOORBY, 1898.* The secular changes of velocity were again found to be great, and it was only after a great number of trials that the following method (suggested by Professor REYNOLDS) was hit upon to reduce them to a minimum, it being the only method suitable for this work.

The boiler was worked at 120 lbs. pressure, and the steam was throttled so as to reach the engine at 5 lbs. per square inch. In this way, small variations of boiler pressure were rendered less effective in causing variations of velocity.

The engines were run so as to give out approximately 20 horse-power, and drove by means of a rope a long line of shafting from which the power was taken to the counter shaft of the machine by means of a 2½-inch belt. The surplus work was dissipated in a hydraulic brake, also described in the paper just referred to. The brake was not loaded in the ordinary way, but was allowed to bed against an upright or dead-stop behind the brake; a fairly constant flow of water was supplied to the brake, and the resistance offered by the brake was varied by regulating the quantity of water passing out of it.

A speed indicator, similar to the one attached to the testing machine, and previously described (see p. 272), was driven directly from the engine, and the heights of the water columns in the two indicators were constantly watched by

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* 'Phil. Trans.'

means of a telescope and mirror, so arranged as to bring the images of the two columns next to each other. The fluctuations of velocity of the engine and the machine could thus be easily compared, and any slipping at once detected.

It was found that, using this method of regulation, the fluctuations of velocity of the machine and engine corresponded with one another, and that the fluctuation could be kept within very small limits, namely, about $\frac{1}{5}$th per cent. A certain amount of experience was necessary to ensure this steady motion for a long period, as in varying the water passing out of the brake, a little too much either one way or the other, oscillations of speed were set up which took some time to die away. It was also found that the reading of the two speed indicators did not correspond at once when the machine was started after a period of rest, but that after a few minutes' run they settled down to corresponding positions.

The telescope and mirror were discarded after some time, but the speed indicators were occasionally checked in each experiment. Mr. JOSEPH HALL, the engine attendant in the Whitworth Engineering Laboratory, soon became quite expert in keeping the variations of speed within surprisingly narrow limits, even when an experiment extended over seven or eight hours without a stop.

The author often found it impossible to be in attendance the whole time occupied by long tests, and in such cases the machinery was left in charge of Mr. HALL. The author found that he could leave the apparatus in his charge with the utmost confidence.

The maximum error in the determination of the stress, due to errors in the measurement of $\omega$, is finally estimated at ·3 per cent.

### *Modes of Vibration of a Specimen.*

The specimen may vibrate during a test in three ways, longitudinally, transversely, and torsionally, and it is important that, either the periods of the free vibration of the specimen do not coincide with the period of any unbalanced force in the machine, or that the vibrations are prevented from taking effect by the use of suitable guides.

The central cylindrical part of most of the specimens was half an inch long and ·25 inch diameter. The greatest load suspended from it was 43·77 lbs., and the smallest 12·42. Taking $30 \times 10^6$ as YOUNG'S modulus for mild steel, the number of longitudinal vibrations per minute was calculated and found to be between the limits 130,000 and 50,000 approximately.

The highest speeds at which the machine was driven with the greatest and least loads were 1,800 and 2,500 revolutions per minute respectively. It is thus evident that the free period of the longitudinal vibration of the specimen can never coincide with, or be any simple multiple of, the speed of the machine, and hence can never coincide with any periodic force arising from the imperfect balancing of the moving parts.

The number of free vibrations per minute of the specimen vibrating transversely was calculated, considering the specimen as a bar fixed at its end, and found to be about 500,000 per minute, so that in this case also the free vibration need not be considered as influencing the results.

The number of vibrations per minute executed by a specimen when oscillating torsionally was also estimated. The calculation gives for the heaviest and lightest loads used 1200 and 2800 vibrations per minute respectively, so that it is quite possible that the free period of torsional vibration of the specimen might coincide with the speed of the machine.

The key attached to the vertical sliding spindle, which works in a key-slotted bush which can be adjusted and locked, prevents such vibration *above a certain amplitude* taking place, and they are damped also by the viscosity of the oil in the bearings of the spindle. Still, it is evident that this key and keyway cannot be fitted so accurately as to completely extinguish a twist in the specimen as it slides with the weight.

With a view to eliminating the effect of the torsional vibrations of specimens, a number of tests were carried out under different conditions. It was found that when the speed corresponding to the free period of torsional vibrations was reached, a change in the moment of inertia of the load seemed at once to eliminate the vibrations, whilst the unlocking of the lower bearing greatly increased them, causing the specimen to break with fewer reversals. When the speed did not correspond with the free period, neither the change of moment of inertia nor the conditions ot locking effected the results (see footnote, Table III., set C).

The results of these tests are given later (p. 289), and it is seen that only when the free vibration of the specimen coincides with the speed of the machine has this vibration any influence on the results.

## *Preparation of the Specimens.*

The materials used in the tests, of which the results are given in this paper, were mild steel, best cast steel, and best Lowmoor iron. By far the greater number of tests were carried out on specimens having dimensions given in fig. 4.

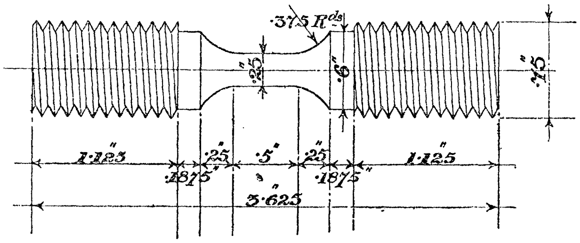

Fig. 4.

Bars $\frac{3}{4}$ inch diameter were cut up into short lengths of 6 inches. The centres were marked and small holes were drilled up these centres for each piece, and the pieces were then square-centred in the ordinary way. A rough cut was then taken over the whole length, and the cutting of the screw, over 5 inches in length, was then commenced. This part was then finished by means of a chaser to a Whitworth Standard $\frac{3}{4}$-inch thread. The pieces were marked off to a standard length and the ends cut down well below the thread. The central part was roughed out and turned carefully as the size required was approached, a template being used for the curved ends, which were cut to an arc of a circle $\frac{3}{4}$ inch diameter.

The specimens were next rotated between centres and ground to size by means of a small emery wheel which rotated rapidly in the opposite direction to the specimen, and which was moved backwards and forwards parallel to the axis of the specimen. The amount taken off during the grinding process was usually about one-thousandth part of an inch all over the central parallel part.

In the early part of the work the specimens were turned, ground, and finished in the College Laboratory, but during the latter part they were prepared by Messrs. CARTERS and Co., Engineers, Salford.

A number of specimens were turned and ground very roughly in order to see the effect of any bad workmanship, but the results were found to agree almost as well as if the specimens had been turned and ground in the careful manner above described.

In the case of the cast-steel specimens the bars were sawn up and the short pieces were then annealed in a gas furnace before commencing the turning process.

## *The Annealing of the Specimens.*

The finished specimens which were to be annealed were placed inside a piece of wrought-iron piping, 6 inches diameter, and the pipe was closed at both ends by means of two cast-iron covers. The case so formed containing the specimens was placed so as to stand with the specimens vertical, inside a gas furnace, and heated.

The jet of hot gases was prevented from playing directly on the case by using a cast-iron plate which was placed opposite to the jet, and the case was rotated frequently to ensure uniform heating. The supply of gas and air to the burner could be adjusted as required. The process of heating up to a red-heat usually took about half an hour, during which time the specimens were occasionally examined by moving the upper cast-iron cover; the gas supply was then diminished so as to keep the furnace at a constant temperature for another half-hour, after which the burner was taken away, the passages for outlet and inlet of hot gases plugged up by pieces of cast-iron, and the whole allowed to cool, the cooling process usually taking from 10 to 12 hours.

On taking the specimens from the annealing furnace the thin coat of oxide was removed from the central parallel part by rubbing it with the finest emery cloth. This coat was, as a rule, easily removed, but in a few cases the specimens were polished in a lathe, as the skin was found to be very hard.

The diameter of the central part was next measured by means of an ordinary micrometer gauge, and if this part was found to be slightly tapered, the diameters at the centre and each end were measured.

### *Method of Fixing the Specimens.*

A lock-nut was screwed on to each end of the specimen, a hardened steel ball, $\frac{1}{2}$ inch diameter, was inserted in the upper chuck H (fig. 5), and that end of the specimen which contained the centre mark was screwed into this chuck, but not screwed home. The oscillatory weight was then next brought up and the chuck J, which contained another hardened steel ball, was screwed on the lower end of the specimen until the specimen bedded against the ball, the specimen being prevented from rotating by means of a pair of gas-tongs with which the short parallel part *m* was gripped. The lock-nut *n* was screwed tight, thus fixing the specimen to the oscillatory weight.

The weight was now supported, and the specimen screwed up so as to bed against the steel ball in the upper chuck, the small force necessary for this being supplied by gas-tongs, with which, in this case, the parallel part *l* was gripped. The lock-nut *r* was then screwed tight, thus fixing the specimen to the chuck H.

By the above method, one was certain of getting the specimen into the machine without straining it, whereas, had the specimen been fixed first to the upper chuck, it is quite possible that the material would have been subjected to severe torsional strains in connecting up the oscillating weight.

In the tests carried out with the apparatus driven by the oil engine, the steel balls were not used, and they were also discarded in the tests on wrought iron and cast steel, since it was found that the specimens could be easily locked in the manner described above without using the balls.

In a great many of the tests the specimens were not prevented from rotating, for, as previously explained, it was found unnecessary to do so, but when necessary, to prevent oscillations, the lower bearing was locked by means of a small brass set-screw *k*, $\frac{1}{4}$ inch diameter, which was screwed so as to press against the outside of the loose bush which formed this bearing, thus by frictional force preventing torsional oscillations.

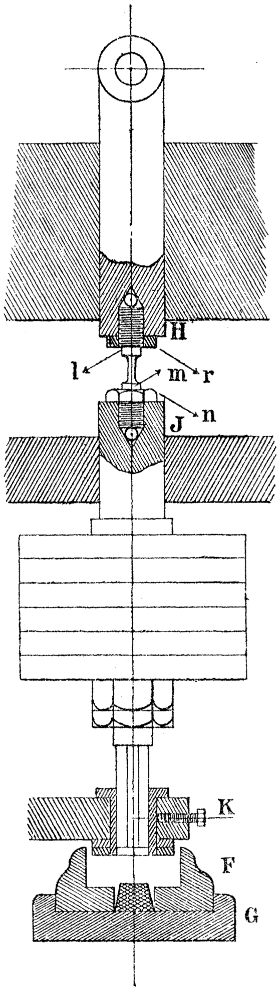

Fig. 5.

### *Method of Conducting Tests.*

The boiler fire was generally made about 8.30 in the morning; steam was up and the engine was started a little before 9.30. During this time the oil supply pipes of the testing machine and engine were attended to, the cocks being adjusted to give the necessary flow for each bearing; the large oil vessels on the wall were filled up; the rope was placed on a suitable step of speed pulley, and the movable pulley was adjusted to give the requisite tension for driving; a specimen was inserted in the manner previously described; the counter reading was taken; the water inlet cock connected to the hydraulic brake was adjusted so as to give the necessary flow of water through the brake; and the apparatus was then ready for the carrying-out of a test.

Having decided upon the particular speed at which the machine was to run, the speed of the engine to obtain this was found from the tabulated results of a series of experiments previously made; the flow of water from the engine brake was regulated (see p. 275) by means of the water outlet valve, attached to the spindle of which was a long arm, which enabled one to delicately adjust the valve opening (so as to bring the engine to the chosen speed); the machine was started after the engine had been working steadily for a few minutes.

At the commencement of a test the author usually watched the speed indicator attached to the machine; the engine attendant watched the engine speed indicator and adjusted the outlet valve of the brake if necessary. The speed of the machine gradually rose; the time taken to rise to the required speed varied from 60 to 100 seconds when the machine was started first thing in the morning, but the speed was attained in about 20 seconds when the machine had been running for a short time before beginning the test.

It was very important to carefully watch the speed indicator columns on starting, as the speed of the engine had to be reduced always one or two revolutions per minute in the first few minutes of the run, which appeared to be the time necessary to obtain steady lubrication of the bearings; moreover, a great amount of trouble was saved when the brake had been carefully adjusted at the commencement of the test, for in many experiments, when this adjustment had been made, it was found unnecessary to touch the outlet cock for periods of 30 minutes or even longer. On arriving at the steady speed, it was only necessary to watch the speed indicator on the engine, since the fluctuations of the two speed indicators were found to agree, but still the indicator connected to the machine was examined about every 10 minutes.

Throughout the test the boiler was attended to, so as to keep the pressure as nearly constant as possible.

The time at which the machine attained the required speed was taken by means of a watch; and when the speed of the engine had become steady, the counter on the testing machine was pushed home, and the number of revolutions taken for from 10 to 100 minutes, according to the length of the time taken for the test; after one minute's interval, during which the counter reading was taken, the counter was again pushed home, and so on throughout the test.

The time at which the specimen broke was taken, and the total time from attaining full speed to breaking was deduced and the total reversals were estimated from this and the mean speed.

In the case of very short tests the speed was found either before or directly after the test, in which case the oscillatory weight was connected up by a specimen of large diameter specially kept for such work, and the machine was run for 10 minutes to the mark on the speed indicator at which the test was done.

It is thus seen that the reversals of stress to which the materials were subjected during the rise of speed were not taken into account. This might make a slight difference in the short tests, but could certainly have very little effect on the results in the case of long tests, namely, those extending over a period greater than two or three hours. To reduce this effect as much as possible, short tests were never carried out without previously running the machine for some time at the speed at which they were done.

If, during a test, the lubrication of any of the bearings of the engine, shafting, or machine failed, it was at once detected, since the speed indicator column on the engine fell and the pitch of the (unmusical) note given out by the machine was lowered. In such cases, the machine was at once stopped and the time taken; the bearing was attended to, and a new start made.

In the case of very long tests, the machine was run for five or six hours each day according to convenience. The engine could not be used for driving on Tuesday or Wednesday afternoons, as it was employed during those times by the students for the experimental trials, which form part of the College Laboratory course; so that a test which required ten or twelve hours to complete could not be conveniently carried out without a stop which, in some cases, extended over two or three days.

It was important to find the effects of these periods of rest on the number of reversals required for rupture, and after several preliminary tests were completed, a series of experiments were undertaken to investigate this matter. An account of these experiments is given later (see p. 283).

### *Preliminary Tests for Mild Steel.*

A great number of preliminary tests were carried out during the time in which the oil-engine was used as the source of power and also on first using the steam-engine. In some cases the oil-engine had an ordinary pulley and cord brake attached, so as to cause the engine to work at full load, thus reducing the fluctuations of velocity, whereas in other cases it was used to drive the machine without any brake.

The ordinary statical test for this material gave the following results:—

| | |
|---|---|
| Yield stress | 18·64 tons. |
| Maximum stress | 25·83 ,, |
| Breaking stress | 22·09 ,, |
| Percentage elongation at maximum stress | 22. |
| ,, ,, rupture | 29. |

The results of the endurance tests given in Table I. were obtained whilst working under the conditions mentioned above. The specimens were turned and finished in the College Laboratory, and approximately corresponded to the final specimen shown in fig. 3. The diameters were not quite the same for all specimens, but varied from ·21 to ·26 inch; the specimens were not annealed. The machine was working with the full load, 43·77 lbs.; the speeds used varied from 1200 to 1500 revolutions per minute.

The fluctuation of velocity, as shown by the speed indicator attached to the machine, usually ranged from 4 per cent. to 6 per cent. The revolutions per minute were determined by dividing the total revolutions by the total time from the beginning to the end of the test.

### TABLE I.—Unannealed Mild Steel.

Oscillatory weight, 43·77 lbs. Diameter of Specimens, ·22 inch to ·26 inch.

| Number. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|
| 1 | 18·97 | −16·36 | 35·33 | 2,360 |
| 2 | 17·94 | 15·48 | 33·42 | 2,330 |
| 3 | 16·43 | 14·17 | 30·60 | 5,960 |
| 4 | 15·99 | 13·80 | 29·79 | 10,240 |
| 5 | 14·82 | 12·90 | 27·72 | 53,200 |
| 6 | 14·73 | 12·72 | 27·45 | 39,700 |
| 7 | 14·19 | 12·19 | 26·38 | 17,200 |
| 8 | 13·68 | 11·87 | 25·55 | 89,200 |
| 9 | 14·09 | 12·15 | 26·24 | 68,900 |
| 10 | 13·67 | 11·73 | 25·40 | 65,400 |
| 11 | 13·36 | 11·39 | 24·75 | 71,400 |
| 12 | 13·24 | 11·43 | 24·67 | 97,800 |
| 13 | 13·08 | 11·36 | 24·44 | 132,000 |
| 14 | 12·41 | 10·70 | 23·11 | 251,000 |
| 15 | 11·97 | 10·30 | 22·27 | 332,000 |
| 16 | 11·76 | 10·01 | 21·77 | 396,000 |
| 17 | 11·72 | 10·01 | 21·73 | 404,000 |
| 18 | 11·57 | 9·91 | 21·48 | 710,000 |
| 19 | 10·42 | 8·84 | 19·26 | 1,930,000<br>(Not broken.) |
| 20 | 11·12 | 9·45 | 20·57 | 3,920,000<br>(Not broken.) |

On comparing these results with those of WÖHLER for a similar material—although it is impossible to choose from his list one exactly the same as the steel used here—one sees the general similarity of the results, but is struck by the great difference between the total reversals for any given range of stress. It is easy to see that this difference is greater as the stress range, and therefore as the speed increases, thus suggesting that there is a relation similar to WÖHLER's for every speed.

If the mean variation of speed is taken at 5 per cent., the variation of stress range due to this will be 10 per cent. This great variation would seem to reduce the value of these tests, but the author introduced them mainly to enable one to compare results got in this way with those obtained after the fluctuations of speed had been reduced to a minimum. In some cases, this fluctuation is of no consequence, as will be seen from the following.

On comparing Tests 18 and 20 (Table I.) we see that for a drop of ·91 ton per square inch in the range of stress, the reversals for rupture have increased from $7{\cdot}1 \times 10^5$ to over $3{\cdot}9 \times 10^6$. If, then, a fluctuation of range of stress of 10 per cent. had been taking place for a test at about this particular range, it is evident that only a very small fraction of the actual reversals recorded could be effective in damaging the material. One thus sees that, as long as the rate of change of reversals with range of stress is small, slight fluctuations of velocity will not appreciably affect the results, whereas, when this rate is great, it is important to keep the speed as steady as possible.

If the limiting range of stress increases as the speed diminishes, it will be more rapidly approached with the method of lowering the range used here, than in that used by WÖHLER, for the diminution of range is got by diminishing the speeds when the specimens are of constant diameter. Hence, after a certain point, it will not be worth while doing long tests, since these fluctuations of velocity, however small, would render the results doubtful.

Finding that the reversals for rupture with any given range of stress are diminished with the speed, the author decided to limit his tests more particularly to cases for which the reversals were less than one million. In a few cases, however, specimens have been subjected to a greater number of reversals.

Nearly the whole of one year was spent in an attempt to get more regular results by improving the method of preparing the specimens, by annealing, by subjecting each specimen to a number of reversals with a small range of stress, and by diminishing the fluctuations of speed in the manner described previously; and, strange as it may seem, the results were not so regular in many of the final series of tests as in those recorded in Table I. The only possible explanation is that the material used for Experiment I. was of more uniform quality than that used subsequently.

### *On the Restoring Effect of a Period of Rest.*

Since in the long tests, namely, those extending over several days, the experiments could not be conveniently carried out without stopping, and therefore allowing the specimen to rest, it is important to find the effect of these periods of rest on the total reversals required for breaking. With this object in view, tests were made with a number of specimens of mild steel, some of which were broken without stopping the machine, while others were allowed to rest for various periods after having been subjected to about half the number of reversals required to break them. The speed of the machine and the suspended load (24·96 lbs.) were kept the same in all cases. The results of these tests are given in Table II.

TABLE II.—Mild Steel.

Oscillatory weight, 24·96. Diameter of Specimens, ·265 inch to ·269 inch.

Unannealed.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals. | Total reversals. |
|---|---|---|---|---|---|---|
| 1 | 1948 | 11·56 | 10·23 | 21·79 | 114,000 | 114,000 |
| 2 | 1949 | 11·35 | 10·06 | 21·41 | 122,000 | 122,000 |
| 3 | 1947 | 11·53 | 10·21 | 21·74 | 124,000 | 124,000 |
| 4 | 1940 | 11·54 | 10·22 | 21·76 | 47,500 | 47,500 |
| 5 | 1947 | 11·69 | 10·37 | 22·06 | 58,400 | — |
| (After 3 days) | 1933 | 11·51 | 10·21 | 21·72 | 53,800 | 112,200 |
| 6 | 1935 | 11·37 | 10·08 | 21·45 | 58,100 | — |
| (After 5 days) | 1938 | 11·41 | 10·11 | 21·52 | 60,200 | 118,300 |
| 7 | 1938 | 11·38 | 10·09 | 21·47 | 58,200 | — |
| (After 5 days) | 1942 | 11·53 | 10·02 | 21·55 | 80,600 | 138,800 |
| 8 | 1937 | 11·49 | 10·17 | 21·66 | 58,200 | — |
| (After 11 days) | 1932 | 11·43 | 10·11 | 21·54 | 58,500 | 116,700 |
| 9 | 1940 | 11·39 | 10·09 | 21·48 | 58,200 | — |
| (After 11 days) | 1936 | 11·37 | 10·07 | 21·44 | 7,500 | 65,700 |
| 10 | 1944 | 11·58 | 10·26 | 21·84 | 58,300 | — |
| (After 4 months) | 1942 | 11·56 | 10·23 | 21·79 | 30,800 | 89,100 |
| 11 | 1944 | 11·57 | 10·24 | 21·81 | 58,300 | — |
| (After 4 months) | 1950 | 11·63 | 10·31 | 21·94 | 32,100 | 90,400 |
| Annealed.<br>Diameter of Specimens, ·25 inch. | | | | | | |
| 12 | 1947 | 13·14 | 11·62 | 24·76 | 30,600 | 30,600* |
| 13 | 1947 | 13·14 | 11·63 | 24·77 | 30,500 | 30,500* |
| 14 | 1947 | 13·09 | 11·58 | 24·67 | 15,600 | — |
| (After 4 months) | 1938 | 12·98 | 11·46 | 24·44 | 20,250 | 35,850 |
| 15 | 1947 | 13·87 | 11·21 | 25·08 | 15,600 | — |
| (After 4 months) | 1936 | 16·40 | 13·35 | 29·75 | 12,600 | 28,200 |

No. 15 had extended during the first part of the test, and the diameter had changed from ·2487 to ·2271.

In these experiments, two sets of specimens were prepared all from the same bar. The material was the same as that employed in the tests recorded in Table III. The first set consisted of 11 specimens, which were tested without annealing. The length of each was ·63 inch, and their diameters varied from ·265 to ·269 inch.

Specimens 1, 2, 3, and 4 were broken without stopping the machine. Specimens

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* These two tests are also included in Table III., Set C, Nos. 37, 38.

5 to 11 were put into the machine and run for 30 minutes each—about half the length of time required to break Specimens 1, 2, and 3. Specimen 5 was carefully put away for three days, after which the test was completed by putting it in the machine and running it till rupture took place. The other specimens were treated in a similar manner, Specimens 6, 7, and 8 resting for five days, Specimens 8 and 9 for eleven days, and Specimens 10 and 11 for four months. The result for Specimen 4 is irregular, and is therefore rejected.

The second set of specimens was treated at a later time. They were annealed before testing. All the specimens of this set had the dimensions shown in fig. 4, which was adopted as the standard size in all succeeding tests.

These results show that, if a specimen is allowed to rest when the test is half completed, there is no appreciable recovery if the period of rest is for a few days only. They suggest that if the period of rest extends over some months the specimens may or may not recover slightly; the extent would appear to depend on the treatment which it has received previous to the test.

To settle definitely the restoring effect of a long period of rest, a great many more experiments would have to be done, but as far as this work is concerned where the specimens were seldom allowed to rest for more than two days, the effect of this rest on the total reversals for rupture is negligible.

### *Relation of Limiting Range to Periodicity of Reversals.*

Under this head are given the results of experiments to determine the variation of the range of stress with speed when the number of reversals for rupture is constant, viz., one million.

Six bars of mild steel were purchased together, and the whole of the specimens for which the results given in Tables II. and III. were obtained were cut from these bars. Six samples, each 18 inches long, were cut, one from each bar, and were tested for statical breaking-stress, &c., in the Owens College Laboratory Testing Machine, which is a Buckton 100-ton machine of the Wicksteed horizontal lever type; the extensions were measured over 8 inches. The figures obtained in these tests were as follow:—

| | Yield-stress. | Maximum stress. | Breaking stress. | Percentage elongation at maximum stress. | Percentage elongation at rupture. |
|---|---|---|---|---|---|
| Maximum . . | 17·44 | 24·70 | 21·08 | 24·6 | 31·5 |
| Minimum . . | 16·81 | 22·93 | 19·34 | 23·0 | 29 |
| Mean . . . | 17·12 | 24·54 | 20·47 | 23·5 | 30 |

Three annealed specimens of the form used in the endurance tests (dimensions according to fig. 4) were also broken in the same testing machine, and the maximum stress measured. The mean for these three tests was 25·81 tons per square inch; the yield-stress and extensions could not be conveniently measured.

During the preliminary tests (described on p. 281), it had been noticed that the reversals for rupture for any given range of stress diminished as the rapidity of reversals increased, and the author decided to carry out a number of tests to investigate this point. He decided to carry out six sets of tests with the standard form of specimen (fig. 4).

In each set of tests, specimens from every one of the six bars were used; the load supported by the specimen was kept constant and the speed varied. The load was changed from set to set so that there might be six tests for any given range of stress, each carried out at a different speed. The first test of each set was, as a rule, done at the highest speed at which the machine could be run. The range of stress was varied for the subsequent tests in the set by reducing the speed. It was decided to limit, in general, the experiments to tests taking not more than two million reversals for rupture. The results of these tests are given in Table III.

### Table III.

### Set A.

### Annealed Mild Steel.

Oscillatory weight, 12·42 lbs. Diameter of Specimens, ·245 inch to ·247 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 1 | 2380 | 10·00 | 8·95 | 18·95 | 28,090 |
| 2 | 2306 | 9·43 | 8·43 | 17·86 | 22,100 |
| 3 | 2240 | 9·01 | 8·05 | 17·06 | 80,700 |
| 4 | 2191 | 8·59 | 7·66 | 16·25 | 136,000 |
| 5 | 2126 | 7·99 | 7·11 | 15·10 | 248,700 |
| 6 | 2047 | 7·51 | 6·67 | 14·18 | 416,000 |
| 7 | 1956 | 6·81 | 6·02 | 12·83 | 334,000 |
| 8 | 1954 | 6·87 | 6·08 | 12·95 | 682,000 |
| 9 | 1909 | 6·51 | 5·74 | 12·25 | 1,138,000 |
| 10 | 1888 | 6·26 | 5·50 | 11·76 | 1,787,000<br>(Not broken.) |
| Unannealed Mild Steel. | | | | | |
| Diameter of Specimens, ·24 inch to ·255 inch. | | | | | |
| 11 | 2492 | 11·83 | 10·65 | 22·48 | 43,000 |
| 12 | 2438 | 11·17 | 10·03 | 21·20 | 34,100 |
| 13 | 2382 | 10·54 | 9·46 | 20·00 | 66,700 |
| 14 | 2356 | 10·09 | 9·02 | 19·11 | 59,000 |
| 15 | 2346 | 9·91 | 8·87 | 18·78 | 61,000 |
| 16 | 2238 | 9·03 | 8·07 | 17·10 | 73,800 |
| 17 | 2190 | 8·33 | 7·44 | 15·77 | 66,000 |
| 18 | 2122 | 8·03 | 7·17 | 15·20 | 226,500 |
| 19 | 2071 | 7·34 | 6·54 | 13·88 | 162,700 |
| 20 | 2015 | 6·72 | 5·96 | 12·68 | 2,025,000<br>(Not broken.) |

TABLE III.—*continued.*

### Set B.

### Annealed Mild Steel.

Oscillatory weight, 18·69 lbs. Diameter of Specimens, ·247 inch to ·249 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 21 | 2285 | 13·86 | 12·39 | 26·25 | 4,100 |
| 22 | 2250 | 13·33 | 11·91 | 25·24 | 6,200 |
| 23 | 2150 | 12·36 | 11·00 | 23·36 | 35,500 |
| 24 | 2170 | 12·29 | 10·96 | 23·25 | 58,900 |
| 25 | 2113 | 11·60 | 10·33 | 21·93 | 86,000 |
| 26 | 2032 | 10·83 | 9·62 | 20·45 | 124,700 |
| 27 | 1934 | 9·88 | 8·74 | 18·62 | 199,700 |
| 28 | 1860 | 9·30 | 7·20 | 17·50 | 350,000 |
| 29 | 1807 | 8·68 | 7·65 | 16·33 | 438,000 |
| 30 | 1790 | 8·77 | 7·71 | 16·48 | 528,000 |
| 31 | 1749 | 8·08 | 7·09 | 15·17 | 843,000 |
| 32 | 1715 | 7·79 | 6·82 | 14·61 | 1,840,000 |
| 33 | 1696 | 7·65 | 6·70 | 14·35 | 5,076,000 (Not broken.) |

### Set C.

### Annealed Mild Steel.

Oscillatory weight, 24·96 lbs. Diameter of Specimens, ·248 inch to ·25 inch.

| | | | | | |
|---|---|---|---|---|---|
| 34 | 2150 | 16·09 | 14·32 | 30·41 | 0 |
| 35 | 2032 | 14·34 | 12·73 | 27·07 | 13,500 |
| 36 | 1962 | 13·52 | 11·97 | 25·49 | 30,900 |
| 37 | 1947 | 13·14 | 11·62 | 24·76 | 30,600* |
| 38 | 1947 | 13·14 | 11·63 | 24·77 | 30,500† |
| 39 | 1903 | 12·62 | 11·13 | 23·75 | 55,250 |
| 40 | 1845 | 11·83 | 10·41 | 22·24 | 143,200 |
| 41 | 1758 | 10·89 | 9·56 | 20·45 | 348,000 |
| 42 | 1693 | 10·10 | 8·82 | 18·92 | 283,000 |
| 43 | 1682 | 9·88 | 8·64 | 18·52 | 783,200 |

### Set D.

### Annealed Mild Steel.

Oscillatory weight, 31·23 lbs. Diameter of Specimens, ·245 inch to ·249 inch.

| | | | | | |
|---|---|---|---|---|---|
| 44 | 1920 | 16·43 | 14·52 | 30·95 | 4,400 |
| 45 | 1887 | 15·74 | 13·89 | 29·63 | 11,090 |
| 46 | 1831 | 15·03 | 13·25 | 28·28 | 14,300 |
| 47 | 1776 | 14·31 | 12·57 | 26·88 | 22,600 |
| 48 | 1729 | 13·31 | 11·66 | 24·97 | 72,700 |
| 49 | 1698 | 12·72 | 11·12 | 23·84 | 92,900 |
| 50 | 1642 | 11·99 | 10·45 | 22·44 | 149,400 |
| 51 | 1609 | 11·63 | 10·12 | 21·75 | 112,000 |
| 52 | 1589 | 11·14 | 9·66 | 20·80 | 400,300 |
| 53 | 1544 | 10·69 | 9·24 | 19·93 | 540,100 |

* Keyed bush of lower bearing free to turn.

† ,, ,, ,, locked.

### Table III.—*continued.*

### Set E.

### Annealed Mild Steel.

Oscillatory weight, 37·50 lbs. Diameter of Specimens, ·244 inch to ·249 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 54 | 1832 | 18·35 | 16·15 | 34·50 | 0 |
| 55 | 1776 | 17·44 | 15·33 | 32·77 | 2,900 |
| 56 | 1689 | 16·04 | 14·05 | 30·09 | 13,900 |
| 57 | 1645 | 14·82 | 12·92 | 27·74 | 20,100 |
| 58 | 1587 | 13·81 | 11·98 | 25·79 | 39,600 |
| 59 | 1583 | 13·78 | 11·95 | 25·73 | 21,400 |
| 60 | 1544 | 13·16 | 11·38 | 24·54 | 46,600 |
| 61 | 1499 | 12·45 | 10·74 | 23·19 | 112,400 |
| 62 | 1475 | 11·63 | 10·02 | 21·65 | 298,200 |
| 63 | 1441 | 11·03 | 9·46 | 20·49 | 650,100 |

### Set F.

### Annealed Mild Steel.

Oscillatory weight, 43·77 lbs. Diameter of Specimens, ·238 inch to ·249 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 64 | 1617 | 17·63 | 15·37 | 33·00 | 16,140 |
| 65 | 1539 | 15·12 | 13·10 | 28·22 | 17,440 |
| 66 | 1491 | 14·05 | 12·10 | 26·15 | 20,150 |
| 67 | 1501 | 13·99 | 12·09 | 26·08 | 45,200 |
| 68 | 1449 | 13·26 | 11·42 | 24·68 | 54,400 |
| 69 | 1401 | 12·50 | 10·69 | 23·19 | 96,600 |
| 70 | 1402 | 12·31 | 10·52 | 22·83 | 273,100 |
| 71 | 1372 | 11·98 | 10·21 | 22·19 | 348,400 |
| 72 | 1345 | 11·43 | 9·73 | 21·16 | 679,000 |
| 73 | 1316 | 11·74 | 9·08 | 19·82 | 542,000 |

### Unannealed Mild Steel.

Diameter of Specimens, ·238 inch to ·250 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 74 | 1660 | 17·28 | 15·10 | 32·38 | 43,000 |
| 75 | 1550 | 14·45 | 12·48 | 26·93 | 73,600 |
| 76 | 1494 | 14·44 | 12·44 | 26·88 | 67,200 |
| 77 | 1462 | 13·94 | 12·00 | 25·94 | 60,600 |
| 78 | 1356 | 12·57 | 10·70 | 23·27 | 199,400 |
| 79 | 1398 | 12·28 | 10·51 | 22·79 | 106,800 |
| 80 | 1399 | 12·17 | 10·41 | 22·58 | 233,200 |
| 81 | 1326 | 11·86 | 10·07 | 21·93 | 448,000 |
| 82 | 1305 | 11·41 | 9·64 | 21·05 | 1,141,000 |

These results are probably not so regular as they might have been with specimens all cut from one bar. The second column of Table III. gives the speed at which the machine was run during the test; the reversals are given in the sixth column. The latter were determined from the mean speed, found from the readings of the revolution counter over times ranging from 10 to 100 minutes, and the total time between starting the machine and rupture.

The following tests carried out as part of Set B, Table III., have not been included in that table :—

| | Reversals. | Speed. |
|---|---|---|
| *a*. | 132,700 | 1855 |
| *b*. | 127,000 | 1855 |
| *c*. | 17,800 | 1777 |
| *d*. | 51,250 | 1737 |
| *e*. | 329,000 | 1775 |

These tests were carried out after Nos. 24, 25, 26, and 27 of Set B. Owing to the specimen used in (*a*) breaking in a shorter time than was expected, the test was repeated with the result (*b*). The lower bearing, which had for tests (*a*) and (*b*) been locked, was now allowed perfect freedom to rotate, and tests (*c*) and (*d*) were carried out. The lower bush was now removed and a new keyway and key were made and very accurately fitted. Then test (*e*) was carried out. The large plate weights on the oscillating spindle were changed for others of smaller diameter but of the same weight, and the remaining tests of Table III., Set B, were completed. It is evident from the above that in the tests (*c*) and (*d*) the free period of torsional oscillation of the specimen corresponded with that of the machine (see p. 276).

In Tables IV., V., and VI. are given the corresponding results obtained for tests for Lowmoor iron, cast-steel, and cast-iron, respectively.

The statical tests for these materials gave the following results :—

| | | Yield stress. | Maximum stress. | Breaking stress. | Percentage elongation at maximum stress. | Percentage elongation at rupture. |
|---|---|---|---|---|---|---|
| Lowmoor iron | Maximum . | 16·46 | 23·58 | 23·08 | 24 | 29·4 |
| | Minimum . | 16·40 | 23·55 | 21·10 | 21 | 27·5 |
| | Mean . . . | 16·43 | 23·56 | 22·27 | 22·8 | 28·5 |
| Cast-steel . . | Maximum . | 40·20 | 60·80 | 60·80 | — | 5·9 |
| | Minimum . | 39·45 | 55·30 | 55·30 | — | 2·5 |
| | Mean . . . | 39·85 | 58·10 | 58·10 | — | 3·8 |

The breaking stress for *annealed* specimens of the type used in the endurance tests was 23·1 tons for Lowmoor iron, and 48 tons for cast-steel. In the case of the cast-iron used, the breaking stress was 9·4 tons. As the specimens used were short, the extensions were not measured.

By far the greater number of specimens broke without any appreciable change in diameter or length. A fair number, however, had their diameters greatly increased, but there does not appear to be any definite connection between these changes of dimensions and the range of stress or speed, as far as the author has observed. In only one test, namely, No. 15, Table II., was a change of diameter observed on stopping the machine *before* rupture. The author is led to believe that the change in dimensions occurs, if at all, just before breaking. In only one case had flaws been observed on stopping the machine before the breaking-point was reached, namely, No. 15, Table IV. It showed most peculiar flaws at both ends.

### TABLE IV.—Annealed Lowmoor Iron.

Set A.

Oscillatory weight, 12·42 lbs. Diameter of Specimens, ·245 inch to ·249 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 1 | 2380 | 10·15 | 9·08 | 19·23 | 33,350 |
| 2 | 2308 | 9·33 | 8·35 | 17·68 | 49,200 |
| 3 | 2298 | 9·53 | 8·51 | 18·04 | 36,200 |
| 4 | 2217 | 8·82 | 7·86 | 16·68 | 43,250 |
| 5 | 2122 | 7·99 | 7·10 | 15·09 | 252,500 |
| 6 | 2038 | 7·28 | 6·47 | 13·75 | 89,700 |
| 7 | 2034 | 7·31 | 6·50 | 13·81 | 192,300 |
| 8 | 1969 | 6·73 | 5·97 | 12·70 | 399,600 |
| 9 | 1893 | 6·23 | 5·50 | 11·73 | 111,600 |
| 10 | 1890 | 6·21 | 5·48 | 11·69 | 1,236,000 |
| Set B. | | | | | |
| Oscillatory weight, 24·96 lbs. Diameter of Specimens, ·245 inch to ·248 inch. | | | | | |
| 11 | 2217 | 13·03 | 11·53 | 24·56 | 16,270 |
| 12 | 2131 | 12·19 | 10·74 | 22·93 | 43,800 |
| 13 | 2066 | 11·54 | 10·17 | 21·71 | 63,000 |
| 14 | 2019 | 10·77 | 9·45 | 20·22 | 85,800 |
| 15 | 1975 | 10·50 | 9·20 | 19·71 | 413,000 |
| 16 | 1917 | 9·76 | 8·52 | 18·24 | 342,000 |
| Set C. | | | | | |
| Oscillatory weight, 31·23 lbs. Diameter of Specimens, ·248 inch to ·25 inch. | | | | | |
| 17 | 1916 | 12·87 | 11·25 | 24·12 | 62,700 |
| 18 | 1836 | 11·77 | 10·25 | 22·02 | 80,200 |
| 19 | 1701 | 10·28 | 8·88 | 19·16 | 243,800 |
| 20 | 1630 | 8·48 | 8·14 | 17·62 | 760,100 |
| Set D. | | | | | |
| Oscillatory weight, 43·77 lbs. Diameter of Specimens, ·248 inch to ·249 inch. | | | | | |
| 21 | 1486 | 13·81 | 11·90 | 25·71 | 54,000 |
| 22 | 1397 | 12·15 | 10·37 | 22·52 | 100,400 |
| 23 | 1367 | 11·69 | 9·96 | 21·65 | 299,500 |

## TABLE V.—Annealed Cast Steel.

### Set A.

Oscillatory weight, 12·42 lbs. Diameter of Specimens, ·245 inch to ·25 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 1 | 2382 | 10·22 | 9·16 | 19·38 | 35,680 |
| 2 | 2313 | 9·35 | 8·38 | 17·73 | 50,900 |
| 3 | 2303 | 9·51 | 8·50 | 18·01 | 36,900 |
| 4 | 2226 | 8·66 | 7·74 | 16·40 | 97,000 |
| 5 | 2216 | 8·94 | 7·97 | 16·91 | 47,700 |
| 6 | 2117 | 7·70 | 6·85 | 14·55 | 273,000 |
| 7 | 2116 | 7·75 | 6·93 | 14·68 | 95,200 |
| 8 | 2034 | 7·21 | 6·40 | 13·61 | 472,000 |
| 9 | 1963 | 6·77 | 6·00 | 12·77 | 402,000 |
| 10 | 1892 | 6·29 | 5·56 | 11·85 | 1,327,000 |

### Set B.

Oscillatory weight, 24·92 lbs. Diameter of Specimens, ·246 inch to ·248 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 11 | 2215 | 13·06 | 11·56 | 24·62 | 37,700 |
| 12 | 2163 | 12·56 | 11·09 | 23·65 | 49,400 |
| 13 | 2076 | 11·46 | 10·07 | 21·53 | 86,700 |
| 14 | 2012 | 10·91 | 9·56 | 20·47 | 129,300 |
| 15 | 1972 | 10·37 | 9·08 | 19·45 | 189,300 |
| 16 | 1917 | 9·95 | 8·70 | 18·65 | 337,500 |
| 17 | 1838 | 9·17 | 7·98 | 17·15 | 687,400 |

### Set C.

Oscillatory weight, 31·23 lbs. Diameter of Specimens, ·248 inch to ·25 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 18 | 1975 | 13·73 | 12·05 | 25·78 | 31,200 |
| 19 | 1841 | 11·98 | 10·44 | 22·42 | 107,400 |
| 20 | 1694 | 10·10 | 8·72 | 18·82 | 341,600 |
| 21 | 1650 | 9·74 | 8·39 | 18·13 | 2,270,000 |

### Set D.

Oscillatory weight, 43·77 lbs. Diameter of Specimens, ·248 inch to ·249 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 22 | 1474 | 13·46 | 11·60 | 25·06 | 92,600 |
| 23 | 1395 | 12·19 | 10·41 | 22·60 | 157,000 |
| 24 | 1363 | 11·59 | 9·89 | 21·48 | 265,400 |
| 25 | 1326 | 11·07 | 9·40 | 20·47 | 718,000 |
| 26 | 1303 | 10·64 | 9·00 | 19·64 | 918,000 |

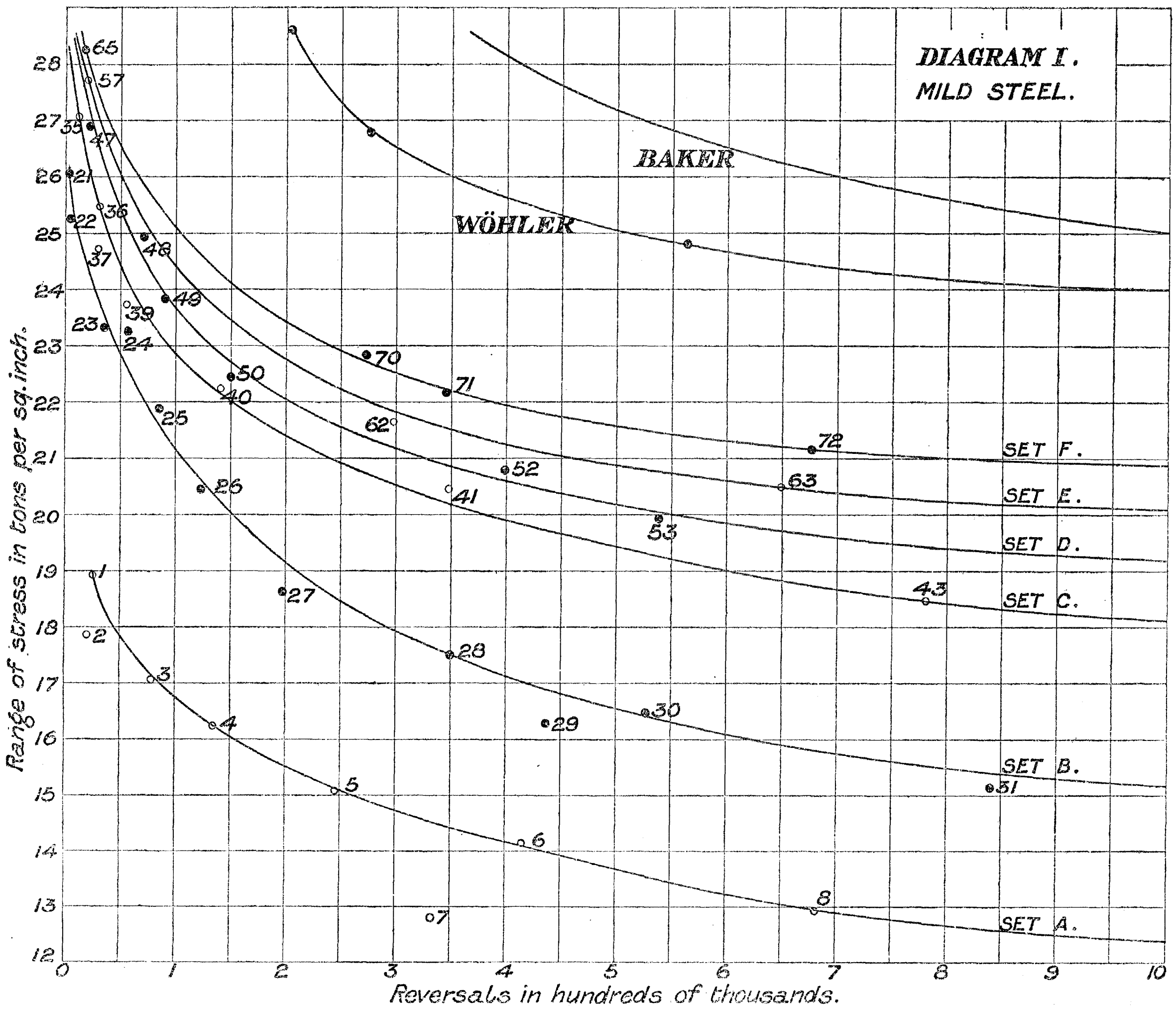
### TABLE VI.—Cast-iron.

Oscillatory weight, 24·96 lbs. Diameter of Specimens, ·345 inch to ·35 inch.

| Number. | Revolutions per minute. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|---|
| 1 | 1941 | 6·67 | 5·91 | 12·58 | 0 |
| 2 | 1493 | 4·01 | 3·46 | 7·47 | 39,200 |
| 3 | 1496 | 4·02 | 3·47 | 7·49 | 36,600 |
| 4 | 1399 | 3·53 | 3·02 | 6·55 | 6,000 |
| 5 | 1396 | 3·51 | 3·01 | 6·52 | 111,600 |
| 6 | 1350 | 3·35 | 2·84 | 6·19 | 0 |
| 7 | 1305 | 3·10 | 2·62 | 5·72 | 620,000 |

### *Summary of Results.*

The results of tests for mild steel, given in Table III., are plotted in Diagram I., the range of stress being taken as ordinate, and the reversals for rupture as abscissa.



The method used was to plot all the results of a set of tests and then draw a curve through the best results, that is, those giving the greatest number of reversals. These are indicated by the small circles in the diagram. To prevent confusion, points lying much below the curves are not shown.

The results obtained by Wöhler and Baker were plotted on the same scale as Diagram I., but on a much larger sheet. It was found that none of Baker's and only three of Wöhler's results came on that part of the sheet included in Diagram II. The three points corresponding to Wöhler's figures will be found on the diagram.

The results chosen from Wöhler's experiments were those carried out on rotating bars (steel axles) made by Messrs. Vickers, Sons & Co. The tenacity of this material ranged from 26·3 to 29·2 tons per square inch, and the percentage extension from 15·8 to 19·5. The material experimented on by Baker was soft steel of tensile strength 26·8 to 28·6 tons, and percentage extension 28.

The results of Wöhler and Baker for their materials are given in the following tables :—

| Number of bar. | Maximum stress. | Minimum stress. | Range of stress. | Repetitions before fracture. |
|---|---|---|---|---|
| Wöhler's. | | | | |
| 63 | 16·3 | −16·3 | 32·6 | 51,240 |
| 64 | 15·3 | 15·3 | 30·6 | 72,940 |
| 65 | 14·3 | 14·3 | 28·6 | 205,800 |
| 66 | 13·4 | 13·4 | 26·8 | 278,740 |
| 67 | 12·4 | 12·4 | 24·8 | 564,900 |
| 68 | 11·5 | 11·5 | 23·0 | 3,275,860 |
| 69 | 10·5 | 10·5 | 21·0 | 8,660,000<br>(Not broken.) |
| Baker's. | | | | |
| 1 | 16·1 | −16·1 | 32·2 | 40,510 |
| 2 | 16·1 | 16·1 | 32·2 | 60,200 |
| 3 | 15·2 | 15·2 | 30·4 | 68,400 |
| 4 | 15·2 | 15·2 | 30·4 | 92,070 |
| 5 | 15·2 | 15·2 | 30·4 | 107,415 |
| 6 | 15·2 | 15·2 | 30·4 | 128,650 |
| 7 | 15·2 | 15·2 | 30·4 | 155,295 |
| 8 | 11·6 | 11·6 | 23·2 | 14,876,432 |

The materials used by Wöhler and Baker in their tests given above, do not correspond very well with that used in the tests carried out by the author, but they are, however, the only results which could be reasonably used for the purposes of comparison.

It appears from Diagram I. that the range of stress for a definite number of reversals diminishes rapidly as the periodicity of the reversals increases. The following Table has been deduced from Diagram I., and shows corresponding values of the range of stress—for rupture at one million reversals—and speed.

Mild Steel.

| Range of stress for rupture with $10^6$ reversals. | Reversals per minute. | Ratio of range for $10^6$ reversals to yield stress. |
|---|---|---|
| 25 (Baker) | 50 to 60 | — |
| 24 (Wöhler) | 60 to 80 | — |
| 20·9 | 1337 | 1·22 |
| 20·1 | 1428 | 1·17 |
| 19·2 | 1516 | 1·12 |
| 18·1 | 1656 | 1·06 |
| 15·2 | 1744 | ·89 |
| 12·4 | 1917 | ·72 |

The author does not consider the number and regularity of the tests on wrought-iron sufficient to enable him to trace the curves for the various loads used, but wishes to point out that the results, with one exception, are similar to, and nearly in agreement with, those obtained for mild steel. The exception is No. 15, Table IV. Finding that the results with wrought-iron were not coming out anything like so regular as in the case of the mild steel specimens, and as the time for the completion of the work was limited, more attention was paid to the cast-steel specimens in the hope (which was realised) of obtaining more uniformity.

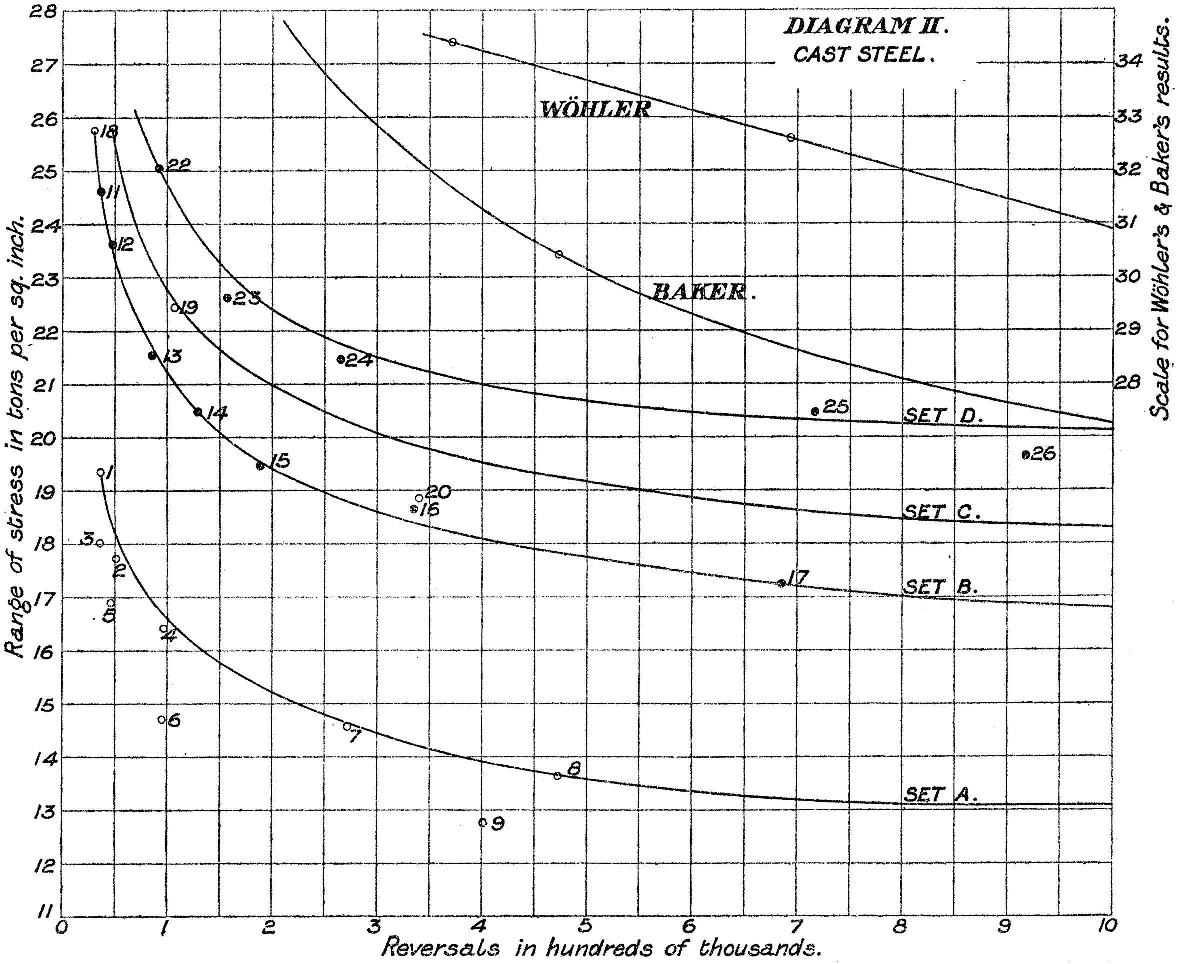
The results of the tests for cast-steel are plotted on Diagram II. They show the same lowering of the range of stress, for any fixed number of reversals for rupture as the speed is increased. The material experimented upon by Wöhler which corresponds most nearly to the cast-steel used here was tool steel made by Firth and Sons, of tensile strength 55 tons, and extension 9·1 per cent. The range for one million reversals, as deduced from Wöhler's results, is 30·9 tons per square inch.

Wöhler's results are for bars rotated and bent :—

| Number of test. | Maximum stress. | Minimum stress. | Range of stress. | Number of repetitions for rupture. |
|---|---|---|---|---|
| 70 | 17·2 | −17·2 | 34·4 | 370,975 |
| 71 | 16·3 | 16·3 | 32·6 | 694,450 |
| 72 | 15·3 | 15·3 | 30·6 | 233,700 |
| 73 | 14·3 | 14·3 | 28·6 | 1,528,550 |

It should be noticed that in these results of Wöhler, the rate of change of reversals with range of stress was *finite* at the point corresponding to one million reversals for rupture. This shows that what is understood as the limiting range was not by any means nearly approached. Diagram II. shows that in these experiments (Table V.) the limiting range for cast-steel was approached very rapidly as the speed was diminished.

It is almost impossible to compare the results obtained here with those given by Baker, as the total reversals in his case were limited to less than half-a-million.



Baker's results for "fine drift steel," of tensile strength 54 tons, elongation 14 per cent., are as follows :—

| Number. | Maximum stress. | Minimum stress. | Range of stress. | Reversals for rupture. |
|---|---|---|---|---|
| 9 | 29·9 | −29·9 | 59·8 | 5,760 |
| 10 | 29·1 | 29·1 | 58·2 | 7,560 |
| 11 | 23·9 | 23·9 | 47·8 | 14,600 |
| 12 | 23·9 | 23·9 | 47·8 | 16,300 |
| 13 | 20·8 | 20·8 | 41·6 | 26,100 |
| 14 | 22·8 | 22·8 | 45·6 | 32,445 |
| 15 | 18·1 | 18·1 | 36·2 | 157,815 |
| 16 | 15·2 | 15·2 | 30·4 | 472,500 |

The range of stress for one million reversals, which these tests of BAKER seem to point to, lies between 27 and 28 tons.

Taking the results given in Table V. along with those of WÖHLER and BAKER, we see that in the case of cast-steel there is a great lowering of the range of stress—for rupture with a given number of reversals—as the speed is increased.

In the following table the author's results are added to those of WÖHLER and BAKER in order to show the relation between the range of stress and the reversals per minute for rupture with one million reversals :—

Cast Steel.

| Range of stress for rupture with $10^6$ reversals. | Reversals per minute. | Ratio of range for $10^6$ reversals to yield stress. |
|---|---|---|
| 30·9 | 60 to 80 (WÖHLER) | — |
| 27·5 | 50 to 60 (BAKER) | — |
| 20·1 | 1320 | ·50 |
| 18·3 | 1660 | ·46 |
| 16·8 | 1820 | ·42 |
| 13·1 | 1990 | ·33 |

In the case of cast-iron the range of stress for one million reversals obtained in the author's experiments is approximately 5·5 tons at 1300 revolutions per minute. WÖHLER obtained 4·78 tons as the range for one million reversals for bars subjected to repeated tensions, the limits being 0 and 4·78 tons; if we assume that cast-iron behaves in the same general way as wrought-iron and steel, WÖHLER'S limit would have been much greater if the stress range had been between equal and opposite limits, pointing possibly to the same lowering of the range as the speed increases.

It is, therefore, impossible, in the case of cast-iron, to say definitely whether the range is diminished as with the other metals experimented upon.

## *Conclusion.*

There are many points which the author would have liked to investigate, but was unable to owing to the great amount of time which would be required. The only satisfactory method of procedure with experiments of the kind dealt with in this paper is to carry out a large number of tests bearing upon any particular point, in order to eliminate the effects of irregularities or inequalities of the materials of which the specimens are composed. It is only in this way that one can be certain of avoiding the inclusion of anomalous results among those from which the deductions are made.

A little time had been spent on the effect of annealing specimens after subjecting them to a number of reversals. The author refrains from publishing the results, which he considers not sufficient to establish anything definite, since they vary a great deal; but desires to mention that the effect of such annealing appeared in general to shorten the life of the specimen and not to restore it, as is usually supposed.

As mentioned early in this paper, complete statical tests of specimens of size shown in fig. 3, could not be performed, since extensions could not be measured; moreover, as the 100-ton testing machine had to be used, the measurements were not too delicate. However, two specimens which had been subjected to reversals, and must have been nearly on the point of fracture, showed a distinctly greater maximum stress than the unused specimens.

The author desires to thank Professor Osborne Reynolds, to whom he is indebted for many suggestions and much valuable advice and criticism during the progress of the work, and also for the facilities which he afforded for the carrying on of the experimental work.